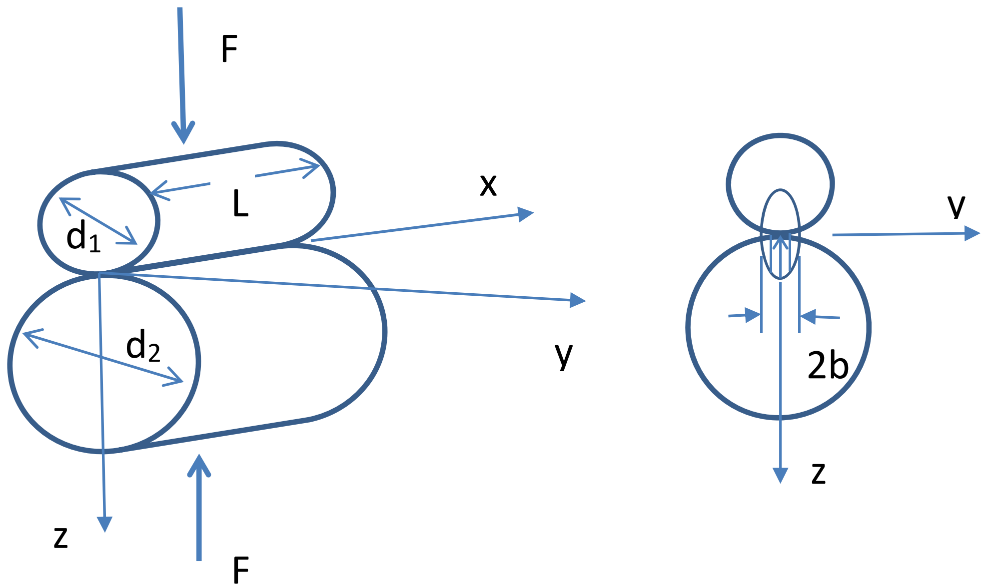
Week 13 Application Activity
ME219 Computer Programming for Engineers

This week we will be revisiting our problem from Week 8, with a few key differences:

a. Stress calculations will be done over a range of distances beneath the surface
b. Maximal shear stress will be calculated (see equation 4)
c. Major program operations will be handled by function files

Consider the diagram below, which depicts two cylinders being held in contact by a force (F) that is uniformly distributed along the length (L) of the cylinders. The resulting stress distribution will be elliptical in shape, such that the maximum stress occurs along the line of contact where $y = z = 0$.



We are interested in calculating the magnitude of the principal normal stresses (σ_x, σ_x, σ_x) and maximum principal shear stress (τ_{max}) in the larger cylinder for points beneath its surface, i.e. for increasing values of z. These may be calculated as follows[1]:

$$\sigma_x = -2\nu_2 p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \frac{z}{b}\right) \quad (1)$$

$$\sigma_y = -p_{max}\left[\left(2 - \frac{1}{1+\frac{z^2}{b^2}}\right)\sqrt{1+\frac{z^2}{b^2}} - 2\frac{z}{b}\right] \quad (2)$$

$$\sigma_z = \frac{-p_{max}}{\sqrt{1+\frac{z^2}{b^2}}} \quad (3)$$

$$\tau_{max} = \frac{\sigma_z - \sigma_y}{2} \quad (4)$$

where b is the half-width of the elliptical distribution, given by

$$b = \sqrt{\frac{2F}{\pi L} \frac{(1-v_1^2)/E_1 + (1-v_2^2)/E_2}{1/d_1 + 1/d_2}} \quad (5)$$

and p_{max} is the maximum pressure, given by

$$p_{max} = \frac{2F}{\pi b L} \quad (6)$$

The following table summarizes the properties of the cylinders we're interested in evaluating.

Symbol	Description	Value
v_1, v_2	Poisson's Ratios for both cylinders	0.3 (both)
E_1, E_2	Young's Modulus for both cylinders	$206 \times 10^9\ N/m^2$
d_1	Diameter of top cylinder	$38\ mm$
d_2	Diameter of bottom cylinder	$70\ mm$
L	Length of both cylinders	$50\ mm$
F	Compressive force	$450\ N$
z	Distance below surface of bottom cylinder	$0 - 0.15\ mm$

Please follow the instructions below to complete this problem.

1. Open MATLAB, navigate to the directory where you want to save your work, and create a new script file called "W13.m".

2. **PART 1**: create variables for the constants in the table above. Note that z will be a vector ranging from 0 to 1.5 mm. Be sure to select an appropriate step size when creating this vector – I suggest 0.001 mm.

3. **PART 2**: Create a function file named "stresscalc.m" to handle your stress calculations. The inputs to this function should include the nine variables listed in the table above; the outputs should include the 4 stresses calculated using equations (1)-(4). In your function file: do the following:
    a. Using equations (5) and (6), calculate b and p_{max}, respectively, and assign them to appropriate variables.
    b. Calculate the following intermediate variables, which will simplify the process of calculating your principal stresses:

$$a_1 = \frac{z}{b} \qquad a_2 = \sqrt{1 + \frac{z^2}{b^2}} \quad (7)\ (8)$$

c. Calculate your 4 principal stress values using equations (1)-(4), using the intermediate variables you defined using equations (7) and (8). Since we're only interested in the magnitude of our stresses, take the absolute value of your resulting stresses.

4. **PART 3**: Create a function file named "stressplot.m" to plot your results. The inputs to this function should include the 4 stresses calculated in "stresscalc", and your z vector. This function will not have any outputs. In your function file, do the following:
    a. Plot your z vector versus each of the 4 stresses on the same plot
    b. Add an appropriate title and axis labels
    c. Add a legend to identify each plotted curve
    d. Put gridlines on your plot

5. **PART 4**: add comments to your MATLAB files that include the following:
    a. A description of what each major section of code does
    b. An description of the stress behavior – what happens to each of the 4 stresses at increasing distances from the cylinder surface?

When finished, please combine all files (W13.m, stresscalc.m, stressplot.m) in a single .zip file named "W13.zip", and upload to Blackboard using the link provided.

REFERENCE

1. Shigley and Mischke, *Mechanical Engineering Design*, 5th ed., McGraw-Hill, New York, 1989.